Practice Test 3 **AP Problems Answers**

1.

$$\int_0^8 \frac{dx}{\sqrt{1+x}} =$$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

2.

If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

3.

Given $f(x) = \begin{cases} x+1 & \text{for } x<0, \\ \cos \pi x & \text{for } x \ge 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2}+\frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}-\frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}+\pi$

4.

$$\int_0^3 |x-1|\,dx =$$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

5.

Let f be a continuous function on the closed interval $[0,2]$. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

6.

If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2 \cos x\,dx$, then $f(x)$ could be

(A) $3x^2$ (B) x^3 (C) $-x^3$ (D) $\sin x$ (E) $\cos x$

7.

An antiderivative of $f(x) = e^{x+e^x}$ is

(A) $\frac{e^{x+e^x}}{1+e^x}$ (B) $(1+e^x)e^{x+e^x}$ (C) e^{1+e^x} (D) e^{x+e^x} (E) e^{e^x}

8.

If $\int_0^k (2kx - x^2)\,dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

9.

$\int_0^{\frac{\pi}{2}} x\cos x\,dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1-\frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2}-1$

10.

If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x, which of the following is NOT necessarily true?

(A) $\int_{-1}^{1} f(x)\,dx > 0$

(B) $\int_{-1}^{1} 2f(x)\,dx = 2\int_{-1}^{1} f(x)\,dx$

(C) $\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$

(D) $\int_{-1}^{1} f(x)\,dx = -\int_{1}^{-1} f(x)\,dx$

(E) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

11.

If $\int_1^4 f(x)\,dx = 6$, what is the value of $\int_1^4 f(5-x)\,dx$?

(A) 6 (B) 3 (C) 0 (D) -1 (E) -6

12.

$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

FRQ 1

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of *f*. Find the *x*-coordinate of the point of tangency, and determine whether *f* has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

(a) $\frac{dy}{dx} = 0$ when $x = 3$

$$\left.\frac{d^2y}{dx^2}\right|_{(3,-2)} = \left.\frac{-y - y'(3-x)}{y^2}\right|_{(3,-2)} = \frac{1}{2},$$

so *f* has a local minimum at this point.

or

Because *f* is continuous for $1 < x < 5$, there is an interval containing $x = 3$ on which $y < 0$. On this interval, $\frac{dy}{dx}$ is negative to the left of $x = 3$ and $\frac{dy}{dx}$ is positive to the right of $x = 3$. Therefore *f* has a local minimum at $x = 3$.

$$3\begin{cases} 1 : x = 3 \\ 1 : \text{local minimum} \\ 1 : \text{justification} \end{cases}$$

(b) $y\,dy = (3-x)\,dx$

$\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C$

$8 = 18 - 18 + C\,;\; C = 8$

$y^2 = 6x - x^2 + 16$

$y = -\sqrt{6x - x^2 + 16}$

$$6\begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } g(6) = -4 \\ 1 : \text{solves for } y \end{cases}$$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

FRQ 2

Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

(a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

(a) $f\left(\frac{1}{2}\right) \approx f(1) + \left(\left.\frac{dy}{dx}\right\vert_{(1,\,0)}\right) \cdot \Delta x$ $= 0 + 1 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$ $f(0) \approx f\left(\frac{1}{2}\right) + \left(\left.\frac{dy}{dx}\right\vert_{\left(\frac{1}{2},\,-\frac{1}{2}\right)}\right) \cdot \Delta x$ $\approx -\frac{1}{2} + \frac{3}{2} \cdot \left(-\frac{1}{2}\right) = -\frac{5}{4}$	$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{answer} \end{cases}$
(b) Since f is differentiable at $x = 1$, f is continuous at $x = 1$. So, $\lim_{x \to 1} f(x) = 0 = \lim_{x \to 1}\left(x^3 - 1\right)$ and we may apply L'Hospital's Rule. $\lim_{x \to 1} \frac{f(x)}{x^3 - 1} = \lim_{x \to 1} \frac{f'(x)}{3x^2} = \frac{\lim_{x \to 1} f'(x)}{\lim_{x \to 1} 3x^2} = \frac{1}{3}$	$2 : \begin{cases} 1 : \text{use of L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$
(c) $\frac{dy}{dx} = 1 - y$ $\int \frac{1}{1-y}\, dy = \int 1\, dx$ $-\ln\lvert 1 - y\rvert = x + C$ $-\ln 1 = 1 + C \Rightarrow C = -1$ $\ln\lvert 1 - y\rvert = 1 - x$ $\lvert 1 - y\rvert = e^{1-x}$ $f(x) = 1 - e^{1-x}$	$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables